mantle model. In order to keep the calculation at an elementary level and to obtain a simple result in closed form, we will use the subsidiary approximation of a constant-gravity mantle (cf. Sec. III). Then the pressure increases from (say) zero at the surface to $D_m \bar{g}_m (R - r_c)$ at the mantle–core interface. It then increases by a further amount $\frac{2}{3}\pi G D_c^2 r_c^2$ in the core itself, so that

$$P_0 = D_m \bar{g}_m (R - r_c) + \tfrac{2}{3}\pi G D_c^2 r_c^2 = 3.4 \times 10^{11} \text{ Pa}, \quad (6)$$

in good agreement with the geophysical value cited above.

[1]S. G. Brush, Am. J. Phys. 48, 705 (1980).
[2]H. Jeffreys, *The Earth* (Cambridge U.P., New York, 1976), 6th ed.

How Olympic records depend on location

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The summer Olympics at Mexico City in 1968 produced a number of anomalous records in track and field, some of which can be explained by the effects of low air density and gravity at Mexico City, while others were the result of outstanding athletic performances.

I. INTRODUCTION

In the study of Newtonian mechanics, students find the standard textbook problems of blocks on inclined planes, etc., to be dull and of little relevance to the "real world" outside the physics classroom. Therefore, it is not surprising that many teachers use athletic events as examples of applications of elementary mechanics in order to generate student interest. An indication of the popularity of applying mechanics to sports is given by the large number of papers in this area which have been published in this Journal and elsewhere in recent years.[1–33]

This paper presents an analysis of several Olympic track and field events with emphasis on the effect of gravity and air resistance on performance. This topic has been extremely well received by students in introductory physics courses at the University of Guelph, and by students in nearby high schools.

II. DATA

Table I presents data on gold medal performances for several Olympic events at sites from Helsinki in 1952 to Los Angeles in 1984. Even a cursory glance will show that there are a number of anomalies in the records for the 1968 Mexico City Olympics. World and Olympic records which still stand unbroken were set in the 400-m dash and the long jump; the performances in the 100-m dash and the triple jump are Olympic records yet. The 200-m dash time remained a world record until 1979, and an Olympic record until 1984. The high jump performance, while not as striking as that in the other events, is still one of only two in the high jump list which was not surpassed at the next Olympics. However, the times for the 5000-m run and the marathon were the worst Olympic times in 16 and 12 yr, respectively.

Students will readily provide explanations for these anomalous records. Because of the high altitude of Mexico City, both the air density and gravity are reduced. The lower gravity produces higher and longer jumps, but has negligible effect on the running events. The reduced air resistance results in enhanced performances in athletic events which are of short duration, such as sprints and jumps. These events are anaerobic in nature: The energy is available from the muscles without the presence of oxygen. However, as the duration of the event increases, the aerobic (oxygen requiring) metabolism provides an increasing fraction of the energy, and the reduced air density becomes an increasing liability because of the lower availability of oxygen. At a time of 20 s into the event, approximately 3% of the energy is being provided aerobically; at 120 s, this has increased to about 75%.[34]

These qualitative explanations, while generally correct, do not, of course, give any indication of the magnitude of the effects of lower air density and gravity. The remainder of the paper considers this topic in some detail; it will become evident that performances in different events are affected to very different degrees by changes in gravity and air density.

III. GRAVITY, AIR DENSITY

The local gravitational acceleration g varies with location because of three major factors: nonsphericity of the Earth, altitude above sea level, and the centrifugal effect. There are additional minor variations due to differences in crustal composition, etc.

The nonsphericity of the Earth and the centrifugal effect produce a sea-level g which varies with latitude, maximum at the poles, and minimum at the equator. The difference between the polar and equatorial g values is 0.18% because of the nonsphericity[35–37] and 0.35% because of the centrifugal effect.[38,39] The effect of altitude on g is well known[40,41]; the decrease in g is roughly 1.97×10^{-3} m/s^2 for every km of land elevation.

Table I. Gold medal performances in various Olympic track and field events.

Location	Year	Men's 100-m dash (s)	Men's 200-m dash (s)	Men's 400-m dash (s)	Men's 5000-m run (min:s)	Men's marathon run (h:min:s)	Men's long jump (m)	Men's triple jump (m)	Men's high jump (m)
Helsinki	1952	10.4	20.7	45.9	14:06.6	2:23:03	7.57	16.22	2.04
Melbourne	1956	10.5	20.6	46.7	13:39.6	2:25:00	7.83	16.35	2.12
Rome	1960	10.2	20.5	44.9	13:43.4	2:15:16	8.12	16.81	2.16
Tokyo	1964	10.0	20.3	45.1	13:48.8	2:12:11	8.07	16.85	2.18
Mexico City	1968	9.95[a]	19.83	43.86[b]	14:05.0	2:20:26	8.90[b]	17.39[a]	2.24
Munich	1972	10.1	20.0	44.7	13:26.4	2:12:20	8.24	17.35	2.23
Montreal	1976	10.06	20.23	44.26	13:24.76	2:09:55	8.35	17.29	2.25
Moscow	1980	10.25	20.19	44.60	13:21.0	2:11:03	8.54	17.35	2.36[a]
Los Angeles	1984	9.99	19.80[a]	44.27	13:05.59[a]	2:09:21[a]	8.54	17.26	2.35

[a] Olympic record, as of 13 August 1984.
[b] Olympic and world record as of 13 August 1984.

Table II presents data on latitude, altitude, and gravitational acceleration[42,43] for the five most recent summer Olympic sites. Because of Mexico City's altitude and proximity to the equator, its gravitational acceleration is the lowest. The largest difference between g values in the table occurs for Moscow and Mexico City (0.37%). In the comparisons of athletic performances to follow, the two sites which are used are Moscow and Mexico City in order to provide the largest difference in g.

The density of the atmosphere decreases exponentially with height[44] for elevations of interest in athletics, with the result that the air density at Mexico City is 22.2% lower than at Moscow—a significant difference.

IV. ANALYSIS OF ATHLETIC PERFORMANCES

The athletic events for which data are presented in Table I fall into three categories: jumps, sprints, and middle or long distance runs.

A. Jumps

1. Long jump

The three jumps (long, triple, and high) are essentially problems in projectile motion with air resistance. For the long jump, the parameters which specify the initial conditions are shown in Fig. 1. The length of the jump is not just the distance $x_{\text{c.m.}}$ traveled by the center of mass (c.m.) while in the air; to this must be added the distances x_T and x_L which represent the horizontal displacements from the center of mass to the foot at takeoff and landing, respectively. Reasonable values are $x_T = 0.25$ m and $x_L = 0.70$ m. The position of the center of mass at landing is below that at takeoff; this difference in vertical displacement was assumed to be 0.40 m. The values of the initial horizontal and vertical components of velocity which were used are $v_{0x} = 9.2$ m/s and $v_{0y} = 3.5$ m/s; these gave a reasonable total distance for the jump and a takeoff angle of 21°, in agreement with observed angles of 18°–22°.[45] The acceleration has a vertical component due to gravity, and both a horizontal and a vertical component due to air resistance.

Because of air resistance, the path of the c.m. while in the air is not a simple parabola, and thus the path was determined numerically. A recent paper[46] in this Journal discusses a number of numerical methods which might be applied to such a problem. The specific approach used in the present work was the half-step approximation, or leapfrog algorithm. The position of the c.m. was calculated at time intervals Δt from the following:

$$v_{1/2,x} = v_{0x} + 1/2 a_{0x} \Delta t, \quad (1)$$

$$x_{n+1} = x_n + v_{n+1/2,x} \Delta t, \quad (2)$$

$$v_{n+1/2,x} = v_{n-1/2,x} + a_{n,x} \Delta t, \quad (3)$$

where, e.g., $v_{1/2,x}$ represents the x component of the velocity at time $t = 1/2\ \Delta t$; a and x represent acceleration and x

Table II. Latitude, altitude, and gravitational acceleration for recent summer Olympic sites.

Year	1968	1972	1976	1980	1984
Site	Mexico City	Munich	Montreal	Moscow	Los Angeles
Latitude (°N)	19.3	48.1	45.4	55.5	34.0
Altitude (m)	2300	525	50	150	50
Measured g (m/s²)	9.7794	9.8072	9.8063	9.8155	9.7958
$g/g_{\text{Mexico City}}$	1.0000	1.0028	1.0028	1.0037	1.0017

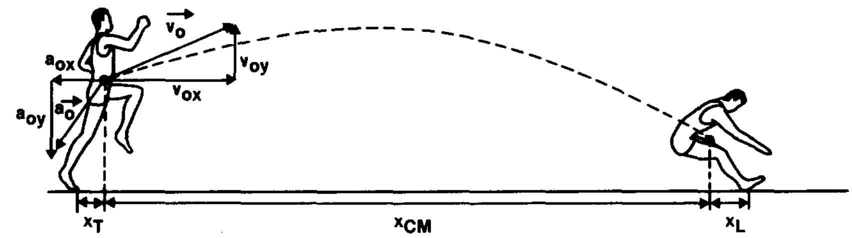

Fig. 1. The long jump.

displacement, respectively; and n refers to the number of time intervals from time $t = 0$. Equation (1) was used only once, to seed the calculation. A similar set of equations was used, of course, in the vertical (y) direction. The acceleration at an integral number of time intervals is needed in Eq. (3), and was determined by considering air resistance and gravity, as follows.

For sizes and speeds of objects moving in athletic events, the force of air resistance (**F**) can be calculated as

$$\mathbf{F} = -k\rho A v^2 \hat{v}, \tag{4}$$

where ρ is the air density, $\hat{v}$ is a unit vector in the direction of the object's velocity with respect to the air, A is the object's projected area perpendicular to this velocity (of magnitude v), and k is a parameter which depends on details of the object's shape. Thus the magnitude of the acceleration due to air resistance can be written (using $a = F/m$) as

$$a = Bv^2, \tag{5}$$

where the parameter B depends on the air density and the jumper's mass, shape, and projected area. Since the acceleration in Eq. (5) depends on the velocity, the determination of acceleration at integral time intervals for Eq. (3) requires a knowledge of velocity at integral time intervals, which can be determined from

$$v_{n,x} = v_{n-1,x} + a_{n-1/2,x}\Delta t \tag{6}$$

and

$$a_{n-1/2,x} = -Bv_{n-1/2}v_{n-1/2,x}. \tag{7}$$

[Similar equations hold in the vertical (y) direction, with gravitational acceleration included.]

Use of Eqs. (1)–(7) allow the calculation of the position and velocity of the c.m. while in the air. A time interval of 1/1000 s was used, which gave results accurate to at least five figures (with double precision arithmetic). For the value of the parameter B, it was assumed that during the first half of the jump the air resistance would be essentially the same as that for a runner, since the jumper is effectively running in the air. Based on actual measurements on runners,[47] B has the approximate value 0.0038 m^{-1} for a runner of mass 65 kg, moving in still air with a density roughly equivalent to that at Moscow. During the second half of the jump, the jumper moves to a squatting position and thus experiences less air resistance. To accommodate this, the value of B in the calculation was decreased in steps during the second half of the jump, with a total reduction of 50%. Because the frontal area of the jumper is not always perpendicular to the velocity during the flight, there is an additional smaller change in B, which was not included.

It was assumed in the calculations that the takeoff velocity was unchanged from Moscow to Mexico City. It is certainly true that a sprinter can run faster in Mexico City because of reduced air resistance, and it might then be argued that the takeoff velocity at Mexico City would be increased. However, it is not clear that jumpers would actually want to run faster in their approaches to the takeoff board. Since the board is only 20-cm wide, the timing, stride length, and rhythm of the approach are very important. Increased speed before hitting the board does not necessarily mean increased takeoff speed; an increase in the approach speed means that the jumper's foot will be in contact with the board for a shorter time, thus making the development of a large vertical velocity component more difficult.

With values for g and B corresponding to Moscow, the total length of the long jump was calculated to be 8.31 m, a reasonable value for a world class jump in 1968, the year of the Mexico City Olympics. With Mexico City g and B, the length was 8.36 m—an increase of 5 cm. Doing a calculation with Mexico City gravity, but Moscow air resistance, results in the conclusion that 47% of the 5-cm increase is due to the decreased gravity at Mexico City and 53% due to decreased air resistance.

The above results indicate that long jumpers at Mexico City received an advantage of at most 5 cm as a result of reduced gravity and air resistance. However, subtracting 5 cm from the gold medal record at Mexico City gives a net jump of 8.85 m, which would still be an Olympic and world record. This result is in agreement with calculations[48–50] done by other authors for this particular event. How was it possible for the Mexico City gold medal winner, Bob Beamon, to achieve such a tremendous jump, when the silver medallist jumped only 8.19 m? The answer is that he simply turned in an exceptional athletic performance; a newspaper account[51] of the event refers to Beamon's "speed" and "height off the board." Beamon simply sprinted hard, and hit the takeoff board well. An increase of only 0.2 m/s in the initial horizontal and vertical velocity components used in the calculations results in a jump the length of Beamon's.

An additional advantage which Beamon experienced was a high tail wind. Many athletic records in jumping and sprinting are approved only if the athlete is aided by a wind of 2.0 m/s or less. The recorded wind speed at the time of Beamon's jump was indeed 2.0 m/s. Redoing the computer calculation with the horizontal component of the jumper's velocity with respect to the air reduced by 2.0 m/s resulted in an additional increase of only 3 cm in the length of the jump. (In Moscow, where a wind would have a larger effect because of the larger air density, the extra advantage of a 2.0-m/s wind was calculated to be 4 cm.) Thus the conclusion that Beamon's record was primarily the result of an outstanding performance is unaltered by including the effect of the wind.

Table III. Takeoff velocities for the phases of the triple jump.

	Vertical component of takeoff velocity (m/s)	Horizontal component of takeoff velocity (m/s)
Hop	2.7	9.5
Step	2.0	8.4
Jump	2.5	6.9

2. *Triple jump*

The triple jump (or hop, step, and jump) can be handled numerically in essentially the same manner as the long jump. For the Moscow calculation, values of initial velocity components (Table III) were used which agree well with measured values,[52] and which produce reasonable distances in the three parts of the jump. In the hop and step it was assumed that the c.m. is 0.08 m lower at landing than at takeoff, and in the jump 0.40 m lower. As in the long jump the total horizontal displacement measured is larger than the distance traveled by the c.m. in the air, because of the position of the c.m. relative to the feet at takeoff and landing. In the three phases of the triple jump, the additional distances which need to be added were estimated to be 1.0, 1.3, and 1.5 m. During the jump phase, the air resistance was reduced by 50% to reflect the squatting position which the jumper assumes before landing.

In the calculation for the triple jump at Mexico City, it was assumed that the takeoff velocity for the hop was the same as that at Moscow. However, a problem arises concerning the values to use for takeoff velocities for the step and jump phases, since the lower air resistance during the jump will result in an increase in these velocities. With normal (Moscow) air resistance the x component of the takeoff velocity in the step is 8.40 m/s, which is 9.7% less than the x component of the landing velocity in the hop (9.30 m/s). With reduced air resistance it was assumed that the percentage reduction is the same, i.e., the x component of the landing velocity at Mexico City (9.35 m/s, calculated in the computer program) is reduced by 9.7% to become the x component of the takeoff velocity (8.44 m/s). Similar reasoning was applied to the y components and to the takeoff velocity for the jump.

The calculations give a triple jump of 17.28 m at Moscow, and 17.41 m at Mexico City—an advantage of 13 cm for the jumper at Mexico City. If the observed jump (17.39 m) at Mexico City is reduced by 13 cm, the new value (17.26 m) would not still be an Olympic record; in fact, it would have been broken at the next Olympics in 1972 with a jump of 17.35 m. It is interesting to note that in the case of the long jump, the Mexico City performance was still outstanding even when the effects of air resistance and gravity are included, but that for the triple jump the performance is reduced to one that might have been expected considering the triple jump data presented in Table I.

About 35% of the increase of 13 cm is due to reduced gravity, while 65% is due to decreased air resistance. This increased importance of the air resistance for the triple jump as compared to the long jump (in which air resistance accounted for 53% of the change) is a reflection of the fact that the triple jumper is in an upright position during most of the jump and squats only at the end, whereas in the long jump the jumper is in a squatting position for almost half the jump.

3. *High jump*

The high jump can be analyzed numerically in a similar way, although the objective here is to find the maximum height of the projectile rather than the horizontal displacement.

Initial horizontal and vertical velocity components were estimated to be 2.5 and 4.0 m/s, respectively. It was assumed that the c.m. of the jumper just clears the bar (although it can be argued that the c.m. of a good jumper might pass under the bar), and that the initial height of the c.m. is 1.4 m. This initial height might seem large, but good high jumpers are tall and long legged. A study of the physiques of Olympic athletes[53] concluded that "high jumpers are tall men ... they have the longest legs relative to the trunk of all the athletes (with the possible exception of the hammer throwers)."

The effect of air resistance on high jumpers is much less than that for long and triple jumpers, because both the speed and the projected area of a high jumper are considerably less. In the back layout technique (also known as the "Fosbury flop" after Dick Fosbury who was the first to use it), the athlete jumps over the bar head first with back down. Thus the projected area is just the top area of the head and shoulders with a small addition due to the legs, which have some bend in the knees. For calculations, the air resistance for a jumper using this technique was estimated to be about 20% of the air resistance of a runner traveling at the same speed.

The results of calculations for the high jump (assuming a back layout) give 2.214 m for Moscow and 2.217 m for Mexico City—a difference of only 3 mm, 94% of which results from the difference in gravity and 6% from the difference in air resistance. High jumps are measured only to the nearest cm, so this difference of 3 mm is negligible. However, the Olympic high jump record set at Mexico City did stand until 1976. Why? The answer lies in the technique itself. Dick Fosbury was the only jumper at Mexico City to use the back layout, which is a superior technique. It was not until Moscow in 1980 that most other jumpers were using it.

B. Sprints

The analysis of the sprints is very different from that of the jumps, since the objective in a sprint is the development of a high speed as quickly as possible. The following analysis uses energy considerations to calculate the speed–time curves for sprints in the case of reduced air resistance.

Figure 2 shows speed–time curves for the 100-, 200-, and 400-m sprints being run in 10, 20, and 45 s, respectively. The 100-m curve up to approximately 9 m/s was based on actual measurements made on sprinters.[54] The maximum speed (12.2 m/s) is slightly less than an estimate of the fastest speed (12.5 m/s) ever attained by a human[55,56]—in 1963 Bob Hayes took 1.1 s to run between the 60 and 75 yard marks in his (then) world record breaking run of 9.1 s for 100 yards. The curves when integrated give distances of 100, 200, and 400 m, and the squares of the curves have been smoothed to ensure that the kinetic energy of the runner increases in a reasonable manner.

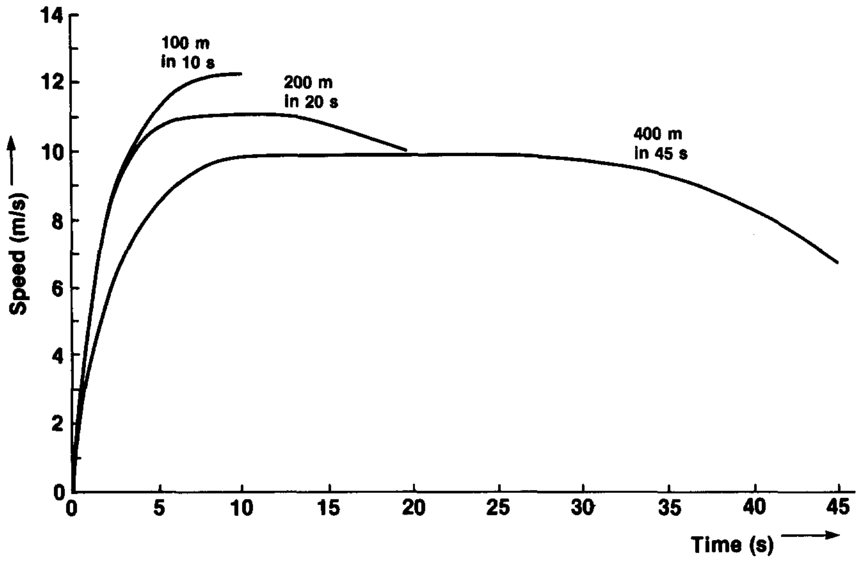

Fig. 2. Speed versus time curves for the sprints.

Although Fig. 2 shows smooth curves, the speed and kinetic energy actually increase in steps which correspond to the strides of the runner. Figure 3 is a representation of the kinetic energy of a runner during one stride; the beginning of the curve corresponds to a time when one foot is just hitting the ground. The kinetic energy then drops quickly as energy E_{brake} is lost to the ground; then it increases by an amount E_{push} as the foot pushes against the ground. During the ensuing glide portion of the stride, the kinetic energy drops slightly because of air resistance (which decreases the kinetic energy while the foot is in contact with the ground as well). E_{air} represents energy lost to air resistance. The increase in kinetic energy ΔKE during the stride is given by

$$\Delta\text{KE} = E_{\text{push}} - E_{\text{brake}} - E_{\text{air}}. \tag{8}$$

The runner is also expending energy in acceleration of his limbs and in production of heat. However, in order to determine the increase in kinetic energy associated with the motion of the c.m., it is necessary only to look at the interaction of the runner with the outside world, namely the ground and the air.

In the case of reduced air resistance at Mexico City, then the energy E_{brake} and E_{push} during a stride will be unchanged (assuming that the foot hits the ground with the same initial speed), but E_{air} will be reduced, thus increasing ΔKE during one stride. We have

$$\Delta\text{KE}_R = \Delta\text{KE} + (E_{\text{air}} - E_{\text{air},R}), \tag{9}$$

where the subscript R refers to reduced air resistance. Equation (9) was used, along with ΔKE and E_{air} as a function of speed, to determine speed versus time curves in the case of reduced air resistance. Rather than being determined stride by stride, the new curves were generated every 0.5 s.

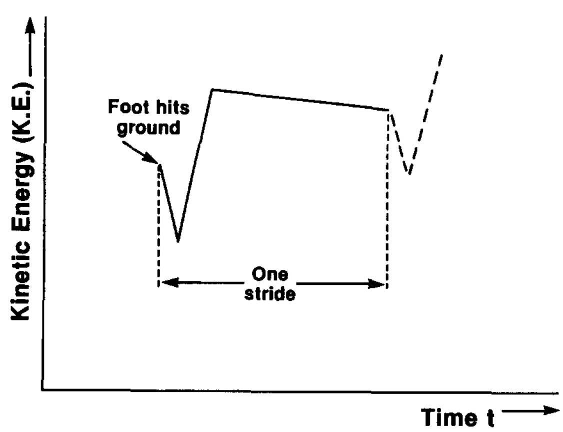

Fig. 3. Kinetic energy of an accelerating runner.

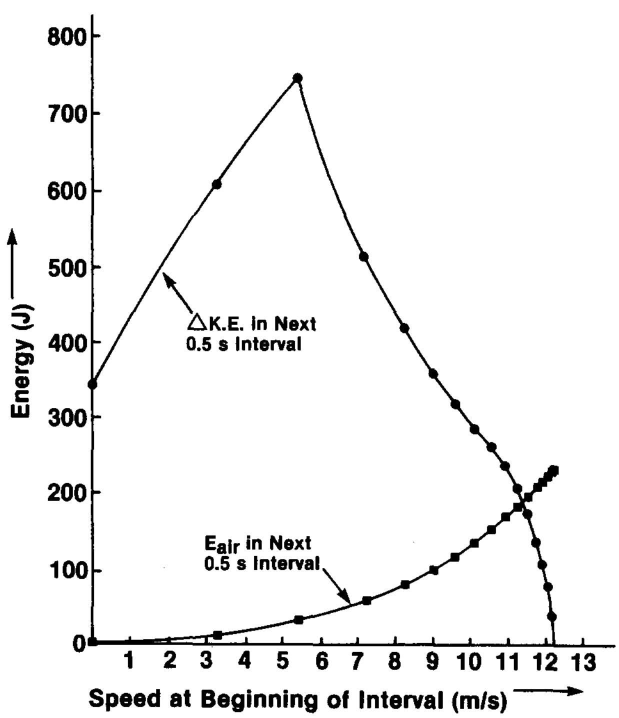

Fig. 4. Energy used against air resistance and increase in kinetic energy, in intervals of 0.5 s for the 100-m sprint.

Figure 4 shows ΔKE and E_{air} for intervals of 0.5 s in the 100-m dash; ΔKE represents the increase in kinetic energy that would occur in the next 0.5 s for a given initial speed, and E_{air} represents the energy lost to air resistance in the next 0.5 s. ΔKE was calculated from the speed data available in Fig. 2 and E_{air} was calculated using the work done by the force of air resistance F; $F = mBv^2$ [from Eq. (5)], where m is the mass of the runner (65 kg assumed). Then $E_{air} = Fd$, where d is the distance run; d can be written as $v\Delta t$, where Δt is the time interval chosen (0.5 s). Thus the final expression for E_{air} is

$$E_{air} = mBv_i^3\Delta t. \tag{10}$$

Since v is not constant during the time interval, the value of v^3 used was $(v_i^3 + v_f^3)/2$, where v_i and v_f are the initial and final speeds for the interval, respectively. Since E_{air} depends linearly on B, which in turn depends linearly on the air density, then $E_{air} - E_{air,R}$ in Eq. (9) can simply be written as $0.778E_{air}$ in the case of the 22.2% reduction in air density between Moscow and Mexico City.

The speed–time curve for the case of reduced air resistance was generated in the following way. Since it is clear from Fig. 4 that air resistance at low speeds (< 6 m/s) is negligible, the first three points (at times 0, 0.5, and 1 s) were assumed to be the same as for the normal air resistance curve. Equation (9) was then used to calculate the change in kinetic energy (and hence the increase in speed) in the next 0.5-s interval. In the case of the 100-m dash, the speed at 1.5 s is thus calculated to be 7.236 m/s at Mexico City, as compared with 7.22 m/s at Moscow. This speed of 7.236 m/s was then used as the initial speed of another 0.5-s interval; ΔKE and E_{air} for this initial speed were taken from Fig. 4 (in actual practice a computer was used), and Eq. (9) was used to calculate the final speed. The complete speed–time curve was generated in this manner. For the production of the high-speed flat portion of the 100-m sprint curve, the ΔKE curve in Fig. 4 was extrapolated (to negative values) beyond the 12.2-m/s limit; this is equivalent to assuming that if the runner's speed were to climb marginally above 12.2 m/s under conditions of normal air resistance it would then in the next 0.5-s interval fall to the stable value of 12.2 m/s again.

The resulting speed–time curve for the 100-m dash in the case of reduced air resistance is only slightly above that for normal air resistance in Fig. 2. The largest difference in speed between the two curves is only 0.24 m/s, at a time of 6.5 s. The maximum speed in the case of reduced air resistance is 12.26 m/s, a very small increment above 12.2 m/s, as would be expected. The resulting time required to run 100 m is 9.91 s, a decrease of 0.09 s from the 10.00 s assumed for Moscow. The time is rather sensitive to the shape of the original v vs t curve; if a curve is used which has a larger maximum speed (12.4 m/s), which must go hand in hand with a somewhat smaller initial acceleration in order to give an integral of 100 m, the time advantage increases to 0.10 s. This value drops to 0.08 s for a curve with a maximum of 12.0 m/s.

If 0.09 s is used as a reasonable correction for 100-m sprinters at Mexico City, the gold medal time of 9.95 s increases to 10.04 s, which is no better than the 10.0 s recorded in Tokyo in 1964.

In the 200-m dash the correction is 0.14 s, again with an uncertainity of 0.01 s resulting from the use of various possible speed curves. The corrected Mexico City time is 19.97 s, which would have remained an Olympic record until 1984. Each of the curves for normal air resistance had a maximum speed (11.0–11.4 m/s) which was somewhat less than the 100-m sprint maximum, followed by a gradual decline in speed over the second 100 m to a final value of 9.75–9.88 m/s. To generate the declining portion of the curve for reduced air resistance, Eq. (9) was used, with ΔKE a negative quantity which represents the loss in kinetic energy in the next 0.5 s, calculated from the speed-time curve for normal air resistance. ΔKE was used here as a function of time, rather than of speed, to reflect the runner's fatigue toward the end of the race, with a resulting loss of kinetic energy which depends on the time elapsed.

The 400-m dash was analyzed in the same manner. However, the advantage at Mexico City calculated in this way will certainly be too high, since after approximately 30 s, one of the anaerobic energy-producing processes begins to "shut down" because of the depletion of phosphocreatine (an energy-storing molecule) in the muscles. This is the reason for the marked decline in the runner's speed after 30 s (Fig. 2). Thus for the last third of the race, the runner will be experiencing the disadvantage of lower oxygen availability at Mexico City. This effect was not included in the calculation, although its magnitude is not as large as might be expected (see Sec. IV C). The calculated advantage for the 400-m dash at Mexico City was 0.33 s; adding this to the actual time of 43.86 s gives a result of 44.19 s, still an Olympic record (and probably a world record too). Thus for both the 200- and 400-m dashes, the lower air resistance does not account for the entire anomaly in the record at Mexico City.

However, it is clear that for the sprints the reduced air resistance at a high-altitude site such as Mexico City provides an advantage which is quite significant. This is emphasized by the fact that the current world record of 9.93 s for the 100-m dash was set at an elevation of 2200 m in Colorado Springs (1983), and the world record of 19.72 s in the 200-m event was set at Mexico City (1979). It is now very difficult for a world or Olympic record in a sprint to be set at a low-altitude site. This applies to other events as well—the world record in the triple jump, 17.89 m, was set at Mexico City (1975). Bert Nelson, editor of *Track and Field News*, has suggested[57–60] that corrections should be applied to events that are held at high altitude, but to date there has been no formal action.

A final point on the topic of sprints concerns the effect of the lower gravity at Mexico City. In the first second or so of the sprint, the runner changes from the crouching position used for the start to an upright position, thus increasing the gravitational potential energy associated with the position of the c.m. This energy, calculated from the standard expression mgh, is 319 J [mass $m = 65$ kg, height $h = 0.5$ m (approximately), and $g = 9.8155$ m/s^2 at Moscow]. At Mexico City, with gravitional acceleration reduced by 0.37%, the value is reduced by only 1 J, which is negligible in comparison with the 950 J of translational kinetic energy which the runner gains in the first second of the sprint. Once the runner has attained an upright stance, the c.m. oscillates up and down with a vertical displacement of about 5 cm (Ref. 1); a change of 0.37% of the gravitational potential energy associated with this motion of the c.m. in one stride is only 0.1 J, again a tiny quantity. Thus the reduced gravitational acceleration at Mexico City has a negligible effect on running events.

C. Middle and long distance runs

Table I presents data concerning a middle distance (5000 m) and a long distance (marathon) run. The time for the 5000-m run in Mexico City is only 3.6% greater than the average time for the 5000 m in the Olympics from 1960–1976 (excluding Mexico City). A similar comparison for the marathon gives a difference of 6.1%. These data are surprising in light of the fact that the oxygen content of the air is reduced by 22% in Mexico City. Experiments[47,61] have shown that the volume of oxygen used per unit time in level running is proportional to the speed. Therefore, it might be expected that an athlete running at his limit of endurance would have to decrease his speed by 22% at Mexico City. Of course, it is true that the air resistance is reduced, but the effect of air resistance is negligible at the speeds of middle and long distance runners (6.1 m/s for 5000 m, 5.3 m/s for the marathon). Even in the 100-m dash, run at much higher speed, the reduced air resistance produces only a 0.9% reduction in the time. The answer to this puzzle appears to be that an athlete who trains at high altitude can acclimatize his body somewhat to the low level of oxygen; this effect is due to changes such as an increase in the number of erythrocytes (red blood cells) in the blood.

V. CONCLUSION

The presentation of the above analysis has been very effective in stimulating student interest and in demonstrating a "real world" application of mechanics. A result of special importance is the fact that the reduced air resistance and gravity at Mexico City can account for some of the anomalous records set there, but in other cases, the records reflect superb athletic performances.

It is clear that high-altitude sites provide a distinct advantage for jumpers and sprinters, and a disadvantage for middle and long distance runners. The day may not be far off in some athletic events when records set at high altitude will not be recognized (in the same way that records set now with an aiding wind of more than 2.0 m/s are not recognized). Alternatively, corrections might be applied based on the results of calculations such as those presented here.

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To derive the existence of gravity

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The existence of gravitational force is derived from simple arguments based on Newton's mechanics. The derivation provides physical understanding both of Einstein's general theory of relativity and its relation to the special theory. In particular, the proportionality between mass and energy is a direct consequence of this derivation.

I. INTRODUCTION

No other physical theory has such a reputation for being difficult to understand as Einstein's general theory of relativity. It is regarded[1] "as the greatest example of the power of speculative thought" and it is often claimed that the theory can be properly understood only by people with a special mathematical education. This impression is reinforced by the fact that for historical reasons, many textbooks have stayed close to Einstein's original derivation. This is not at all necessary and after reordering a few steps in the usual derivation one can explain the basic ideas without using any knowledge beyond Newton's mechanics.

The derivation of Newton's law of gravitation presented here was given as a single lecture to provide students with an understanding of Einstein's theory and to help them visualize the mathematics. The paper is mainly written on a level corresponding to this aim. We have also included some of the historical quotations used in the lecture, since they seem to provide an effective background. Readers mainly interested in the physical argument may go directly to Secs. V and VI.

II. HISTORICAL BACKGROUND

The real triumph of Einstein's theory is that it *derives* Newton's law of gravitation. In the textbooks this fact is often obscured by detailed descriptions of effects that go beyond Newtonian theory, such as the deflection of light in a gravitational field or the precession of the perihelion of planetary orbits.

The origin of gravitation was a matter of great concern to Newton, as can be seen in *Principia*[2]: "Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and the planets, without suffering the least diminution of its force;...and in receding from the sun decreases accurately as the inverse square of the distance as far as the orbit of Saturn, as evidently appears from the quiescence of the aphelion of the planets;...But hitherto I have not been able to discover the cause of those properties of gravity from phenomena and I frame no hypotheses;." The last line is famous and suggests that Newton was dissatisfied with those theories he had actually proposed to himself.

When Einstein finally derived the law of gravitation from other physical principles in 1915, he was mainly inspired by dissatisfaction with his own special theory of relativity. That theory was limited to coordinate frames moving with constant velocity and could not give the energy of a gravitational field. In Einstein's own words[3]: "Suddenly a thought struck me: If a man falls freely, he would not feel his own weight... . I continued my thought: A falling man is accelerated. Then what he feels and judges is happening in the accelerated frame of reference. I decided to extend the theory of relativity to the reference frame with acceleration. I felt that in doing so I could solve the problem of gravity at the same time." Here is clearly seen why Einstein thought of his theory as an extension to a more general theory.

In the present paper we instead start directly from the concepts of the general theory, which seems to be a simplification from the pedagogical point of view. Einstein's analysis of accelerated coordinate systems led rather directly to the idea that space-time is curved and that curvature is the origin of gravitation. The analysis became mathematically complex, however, since a general description of coordinate transformations in space-time had to be introduced at an early stage.[4]

If we instead start directly from the idea that space and time are inhomogeneous, it can be shown that gravity is a consequence of that assumption. The derivation becomes particularly simple when the system may be described by

